

# Package: ddtlcm (via r-universe)

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**Type** Package

**Title** Latent Class Analysis with Dirichlet Diffusion Tree Process  
Prior

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**Description** Implements a Bayesian algorithm for overcoming weak separation in Bayesian latent class analysis. Reference: Li et al. (2023) <[arXiv:2306.04700](https://arxiv.org/abs/2306.04700)>.

**Depends** R(>= 4.3)

**Imports** ape (>= 5.6-2), data.table (>= 1.14.4), extraDistr (>= 1.9.1), ggplot2 (>= 3.4.0), ggpubr (>= 0.6.0), ggtext (>= 0.1.2), ggtree (>= 3.4.0), label.switching (>= 1.8), matrixStats (>= 0.62.0), methods (>= 4.2.3), phylobase (>= 0.8.10), poLCA (>= 1.6.0.1), testthat (>= 3.1.7), truncnorm (>= 1.0-8), BayesLogit (>= 2.1), Matrix (>= 1.5-1), Rdpack (>= 2.5), R.utils (>= 2.12.2)

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**BugReports** <https://github.com/limengbinggz/ddtlcm/issues>

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<code>add_leaf_branch</code>	<i>Add a leaf branch to an existing tree <code>tree_old</code></i>
------------------------------	--

---

**Description**

Add a leaf branch to an existing tree `tree_old`

**Usage**

`add_leaf_branch(tree_old, div_t, new_leaf_label, where, position)`

**Arguments**

<code>tree_old</code>	the original "phylo" tree (with K leaves) to which the leaf branch will be added
<code>div_t</code>	divergence time of the new branch
<code>new_leaf_label</code>	the label of the newly added leaf
<code>where</code>	node name of to which node in the existing tree the new leaf branch should connect to
<code>position</code>	the numerical location of the left side of the added branch

**Value**

a "phylo" tree with K+1 leaves

---

add\_multichotomous\_tip

*Add a leaf branch to an existing tree tree\_old to make a multichotomus branch*

---

### Description

Add a leaf branch to an existing tree tree\_old to make a multichotomus branch

### Usage

```
add_multichotomous_tip(tree_old, div_t, new_leaf_label, where)
```

### Arguments

tree_old	the original "phylo" tree (with K leaves) to which the leaf branch will be added
div_t	divergence time of the new branch
new_leaf_label	the label of the newly added leaf
where	node name of to which node in the existing tree the new leaf branch should connect to

### Value

a "phylo" tree with K+1 leaves that could possibly be multichotomus

---

add\_one\_sample

*Functions to simulate trees and node parameters from a DDT process. Add a branch to an existing tree according to the branching process of DDT*

---

### Description

Functions to simulate trees and node parameters from a DDT process. Add a branch to an existing tree according to the branching process of DDT

### Usage

```
add_one_sample(tree_old, c, c_order, theta, alpha)
```

**Arguments**

tree_old	a "phylo" object. The tree (K leaves) to which a new branch will be added.
c	hyparameter of divergence function $a(t)$
c_order	equals 1 (default) or 2 to choose divergence function $a(t) = c/(1-t)$ or $c/(1-t)^2$ .
alpha, theta	hyparameter of branching probability $a(t) = \Gamma(m-\alpha) / \Gamma(m+1+\theta)$ Allowable range: $0 \leq \alpha \leq 1$ , and $\alpha \geq -2$ beta For DDT, $\alpha = \theta = 0$ . For general multifurcating tree from a Pitman-Yor process, specify positive values to alpha and theta. It is, however, recommended using $\alpha = \theta = 0$ in inference because multifurcating trees have not been tested rigorously.

**Value**

a "phylo" object. A tree with K+1 leaves. if  $t_2 > t_1$ , then select which path to take, with probability proportional to the number of data points that already traversed the path

---

add_root	<i>Add a singular root node to an existing nonsingular tree</i>
----------	---

---

**Description**

Add a singular root node to an existing nonsingular tree

**Usage**

```
add_root(tree_old, root_edge_length, root_label, leaf_label)
```

**Arguments**

tree_old	the original nonsingular "phylo" tree
root_edge_length	a number in (0, 1) representing the distance between the new and the original root nodes
root_label	a character label of the new root node
leaf_label	a character label of the leaf node

**Value**

a singular "phylo" tree

---

attach\_subtree      *Attach a subtree to a given DDT at a randomly selected location*

---

### Description

Attach a subtree to a given DDT at a randomly selected location

### Usage

```
attach_subtree(
  subtree,
  tree_kept,
  detach_div_time,
  pa_detach_node_label,
  c,
  c_order = 1,
  theta = 0,
  alpha = 0
)
```

### Arguments

subtree	subtree to attach to tree_kept
tree_kept	the tree to be attached to
detach_div_time	divergence time of subtree when it was extracted from the original tree
pa_detach_node_label	label of the parent node of the detached node
c	hyparameter of divergence function $a(t)$
c_order	equals 1 (default) or 2 to choose divergence function
alpha, theta	hyparameter of branching probability $a(t) \Gamma(m-\alpha) / \Gamma(m+1+\theta)$ For DDT, $\alpha = \theta = 0$ . For general multifurcating tree from a Pitman-Yor process, specify positive values to alpha and theta. It is, however, recommended using $\alpha = \theta = 0$ in inference because multifurcating trees have not been tested rigorously.

### See Also

Other sample trees: [random\\_detach\\_subtree\(\)](#), [reattach\\_point\(\)](#)

---

`a_t_one`*Compute divergence function*

---

**Description**

Compute value, cumulative hazard, and inverse for divergence function  $a(t) = c/(1 - t)$

**Usage**`a_t_one(c, t)``a_t_one_cum(c, t)``A_t_inv_one(c, y)`**Arguments**

<code>c</code>	a positive number for the divergence hyperparameter. A larger value implies earlier divergence on the tree
<code>t</code>	a number in the interval (0, 1) indicating the divergence time
<code>y</code>	a positive number to take inverse

**Value**

The value and cumulative hazard return a positive number. The inverse function returns a number in the interval (0, 1).

**Functions**

- `a_t_one()`: value of the divergence function
- `a_t_one_cum()`: cumulative hazard function
- `A_t_inv_one()`: inverse function

**See Also**

Other divergence functions: [a\\_t\\_two\(\)](#)

**Examples**

```
a_t_one(1, 0.5)
a_t_one_cum(1, 0.5)
A_t_inv_one(1, 2)
```

---

`a_t_two`*Compute divergence function*

---

**Description**

Compute value, cumulative hazard, and inverse for divergence function  $a(t) = c/(1 - t)^2$

**Usage**`a_t_two(c, t)``a_t_two_cum(c, t)``A_t_inv_two(c, y)`**Arguments**

<code>c</code>	a positive number for the divergence hyperparameter. A larger value implies earlier divergence on the tree
<code>t</code>	a number in the interval (0, 1) indicating the divergence time
<code>y</code>	a positive number to take inverse

**Value**

The value and cumulative hazard return a positive number. The inverse function returns a number in the interval (0, 1).

**Functions**

- `a_t_two()`: value of the divergence function
- `a_t_two_cum()`: cumulative hazard function
- `A_t_inv_two()`: inverse function

**See Also**

Other divergence functions: [a\\_t\\_one\(\)](#)

**Examples**

```
a_t_two(1, 0.5)
a_t_two_cum(1, 0.5)
A_t_inv_two(1, 2)
```



---

compute_IC	<i>Compute information criteria for the DDT-LCM model</i>
------------	---

---

**Description**

Compute information criteria for the DDT-LCM model, including the Widely Applicable Information Criterion (WAIC), and Deviance Information Criterion (DIC). WAIC and DIC are computed using two different methods described in Gelman, Hwang, and Vehtari (2013), one based on (1) posterior means and the other based on (2) posterior variances.

**Usage**

```
compute_IC(result, burnin = 5000, ncores = 1L)
```

**Arguments**

result	a "ddt_lcm" object
burnin	an integer specifying the number of burn-in iterations from MCMC chain
ncores	an integer specifying the number of cores to compute marginal posterior log-likelihood in parallel

**Value**

a named list of the following elements

WAIC\_result a list of WAIC-related results computed using the two methods

DIC1 DIC computed using method 1.

DIC2 DIC computed using method 2.

**Examples**

```
data(result_diet_1000iters)
IC_result <- compute_IC(result = result_diet_1000iters, burnin = 800, ncores = 1L)
```

---

create_leaf_cor_matrix	<i>Create a tree-structured covariance matrix from a given tree</i>
------------------------	---

---

**Description**

Retrieve the covariance matrix of leaf nodes of a DDT tree

**Usage**

```
create_leaf_cor_matrix(tree_phylo4d)
```

**Arguments**

tree\_phylo4d a "phylo4d" object

**Value**

a K by K covariance matrix

**Examples**

```
# load the MAP tree structure obtained from the real HCHS/SOL data
data(data_synthetic)
# extract elements into the global environment
list2env(setNames(data_synthetic, names(data_synthetic)), envir = globalenv())
create_leaf_cor_matrix(tree_with_parameter)
```

---

data_synthetic	<i>Synthetic data example</i>
----------------	-------------------------------

---

**Description**

This list contains one synthetic data with  $K = 3$  latent classes. The elements are as follows:

**Usage**

```
data(data_synthetic)
```

**Format**

A list with 8 elements

**Details**

- tree\_phylo. A "phylo" tree with  $K = 3$  leaves.
- class\_probability. A  $K$ -vector with entries between 0 and 1.
- item\_membership\_list. A list of  $G = 7$  elements, where the  $g$ -th element contains the indices of items belonging to major food group  $g$ .
- item\_name\_list. A list of  $G = 7$  elements, where the  $g$ -th element contains the item labels of items in major food group  $g$ , and the name of the  $g$ -th element is the major food group label.
- Sigma\_by\_group. A  $G$ -vector greater than 0. The group-specific diffusion variances.
- response\_matrix. A binary matrix with  $N = 100$  rows and  $J = 80$  columns. Each row contains a multivariate binary response vector of a synthetic individual.
- response\_prob. A  $K$  by  $J$  probability matrix. The  $k$ -th row contains the item response probabilities of class  $k$ .
- tree\_with\_parameter. A `phylobase::phylo4d` object. Basically the `tree_phylo` embedded with additional `logit(response_prob)` at the leaf nodes.

---

ddtlcm_fit	<i>MH-within-Gibbs sampler to sample from the full posterior distribution of DDT-LCM</i>
------------	--

---

### Description

Use DDT-LCM to estimate latent class and tree on class profiles for multivariate binary outcomes.

### Usage

```
ddtlcm_fit(
  K,
  data,
  item_membership_list,
  total_iters = 5000,
  initials = list(),
  priors = list(),
  controls = list(),
  initialize_args = list(method_lcm = "random", method_dist = "euclidean", method_hclust
    = "ward.D", method_add_root = "min_cor", alpha = 0, theta = 0)
)
```

### Arguments

K	number of classes (integer)
data	an NxJ matrix of multivariate binary responses, where N is the number of individuals, and J is the number of granular items
item_membership_list	a list of G elements, where the g-th element contains the column indices of data corresponding to items in major group g, and G is number of major item groups
total_iters	number of posterior samples to collect (integer)
initials	a named list of initial values of the following parameters: <ul style="list-style-type: none"> <li>tree_phylo4d a phylo4d object. The initial tree have K leaves (labeled as "v1" through "vK"), 1 singleton root node (labeled as "u1"), and K-1 internal nodes (labeled as "u1" through <math>u_{K-1}</math>). The tree also contains parameters for the leaf nodes and the root node (which equals 0). The parameters for the internal nodes can be NAs because they will not be used in the algorithm.</li> <li>response_prob a K by J matrix with entries between 0 and 1. The initial values for the item response probabilities. They should equal to the expit-transformed leaf parameters of tree_phylo4d.</li> <li>class_probability a K-vector with entries between 0 and 1. The initial values for the class probabilities. Entries should be nonzero and sum up to 1, or otherwise will be normalized</li> </ul>

	<p><code>class_assignments</code> a N-vector with integer entries from 1, ..., K. The initial values for individual class assignments.</p> <p><code>Sigma_by_group</code> a G-vector greater than 0. The initial values for the group-specific diffusion variances.</p> <p><code>c</code> a value greater than 0. The initial values for the group-specific diffusion variances.</p> <p>Parameters not supplied with initial values will be initialized using the <code>initialize</code> function with arguments in <code>initialize_args</code>.</p>
<code>priors</code>	<p>a named list of values of hyperparameters of priors. See the function <code>initialize</code> for explanation.</p> <p><code>shape_sigma</code> a G-vector of positive values. The g-th element is the shape parameter for the inverse-Gamma prior on diffusion variance parameter <math>\sigma_g^2</math>. Default is <code>rep(2, G)</code>.</p> <p><code>rate_sigma</code> a G-vector of positive values. Rate parameter. See above. Default is <code>rep(2, G)</code>.</p> <p><code>prior_dirichlet</code> a K-vector with entries positive entries. The parameter of the Dirichlet prior on class probability.</p> <p><code>shape_c</code> a positive value. The shape parameter for the Gamma prior on divergence function hyperparameter c. Default is 1.</p> <p><code>rate_c</code> a positive value. The rate parameter for c. Default is 1.</p> <p><code>a_pg</code> a positive value. The scale parameter for the generalized logistic distribution used in the augmented Gibbs sampler for leaf parameters. Default is 1, corresponding to the standard logistic distribution.</p>
<code>controls</code>	<p>a named list of control variables.</p> <p><code>fix_tree</code> a logical. If TRUE (default), the tree structure will be sampled in the algorithm. If FALSE, the tree structure will be fixed at the initial input.</p> <p><code>c_order</code> a numeric value. If 1, the divergence function is <math>a(t) = c/(1 - t)</math>. If 2, the divergence function is <math>a(t) = c/(1 - t)^2</math>.</p>
<code>initialize_args</code>	<p>a named list of initialization arguments. See the function <code>initialize</code> for explanation.</p>

## Value

an object of class "ddt\_lcm"; a list containing the following elements:

`tree_samples` a list of information of the tree collected from the sampling algorithm, including: `accept`: a binary vector where 1 indicates acceptance of the proposal tree and 0 indicates rejection. `tree_list`: a list of posterior samples of the tree. `dist_mat_list`: a list of tree-structured covariance matrices representing the marginal covariances among the leaf parameters, integrating out the internal node parameters and all intermediate stochastic paths in the DDT branching process.

`response_probs_samples` a `total_iters` x K x J array of posterior samples of item response probabilities

`class_probs_samples` a K x `total_iters` matrix of posterior samples of class probabilities

`Z_samples` a  $N \times \text{total\_iters}$  integer matrix of posterior samples of individual class assignments  
`Sigma_by_group_samples` a  $G \times \text{total\_iters}$  matrix of posterior samples of diffusion variances  
`c_samples` a  $\text{total\_iters}$  vector of posterior samples of divergence function hyperparameter  
`loglikelihood` a  $\text{total\_iters}$  vector of log-likelihoods of the full model  
`loglikelihood_lcm` a  $\text{total\_iters}$  vector of log-likelihoods of the LCM model only  
`setting` a list of model setup information, including: `K`, `item_membership_list`, and `G`  
`controls` a list of model controls, including: `fix_tree`: FALSE to perform MH sampling of the tree, TRUE to fix the tree at the initial input. `c_order`: a numeric value of 1 or 2 (see Arguments))  
`data` the input data matrix

### Examples

```

# load the MAP tree structure obtained from the real HCHS/SOL data
data(data_synthetic)
# extract elements into the global environment
list2env(setNames(data_synthetic, names(data_synthetic)), envir = globalenv())
# run DDT-LCM
result <- ddtlcm_fit(K = 3, data = response_matrix, item_membership_list, total_iters = 50)
  
```

---

<code>div_time</code>	<i>Sample divergence time on an edge <math>uv</math> previously traversed by <math>m(v)</math> data points</i>
-----------------------	--

---

### Description

Sample divergence time on an edge  $uv$  previously traversed by  $m(v)$  data points

### Usage

```
div_time(t_u, m_v, c, c_order = 1, alpha = 0, theta = 0)
```

### Arguments

<code>t_u</code>	a number in the interval (0, 1) indicating the divergence time at node $u$
<code>m_v</code>	an integer for the number of data points traversed through node $v$
<code>c</code>	a positive number for the divergence hyperparameter. A larger value implies earlier divergence on the tree
<code>c_order</code>	equals 1 if using divergence function $a(t) = c/(1 - t)$ , or 2 if $a(t) = c / (1-t)^2$ . Default is 1
<code>alpha, theta</code>	hyperparameter of branching probability $a(t) \text{Gamma}(m-\alpha) / \text{Gamma}(m+1+\theta)$ For DDT, $\alpha = \theta = 0$ . For general multifurcating tree from a Pitman-Yor process, specify positive values to $\alpha$ and $\theta$ . It is, however, recommended using $\alpha = \theta = 0$ in inference because multifurcating trees have not been tested rigorously.

**Value**

a number in the interval (0, 1)

---

draw_mnorm	<i>Efficiently sample multivariate normal using precision matrix from <math>x \sim N(Q^{-1}a, Q^{-1})</math>, where <math>Q^{-1}</math> is the precision matrix</i>
------------	---

---

**Description**

Efficiently sample multivariate normal using precision matrix from  $x \sim N(Q^{-1}a, Q^{-1})$ , where  $Q^{-1}$  is the precision matrix

**Usage**

```
draw_mnorm(precision_mat, precision_a_vec)
```

**Arguments**

precision\_mat precision matrix Q of the multivariate normal distribution  
 precision\_a\_vec a vector a such that the mean of the multivariate normal distribution is  $Q^{-1}a$

---

expit	<i>The expit function</i>
-------	---------------------------

---

**Description**

The expit function:  $f(x) = \exp(x) / (1 + \exp(x))$ , computed in a way to avoid numerical underflow.

**Usage**

```
expit(x)
```

**Arguments**

x a value or a numeric vector between 0 and 1 (exclusive)

**Value**

a number or real-valued vector

**Examples**

```
expit(0.2)
expit(c(-1, -0.3, 0.6))
```

---

exp_normalize	<i>Compute normalized probabilities: <math>\exp(x_i) / \sum_j \exp(x_j)</math></i>
---------------	--

---

**Description**

Compute normalized probabilities:  $\exp(x_i) / \sum_j \exp(x_j)$

**Usage**

exp\_normalize(x)

**Arguments**

x                    a number or real-valued vector

**Value**

a number or real-valued vector

---

H_n	<i>Harmonic series</i>
-----	------------------------

---

**Description**

Harmonic series

**Usage**

H\_n(k)

**Arguments**

k                    a positive integer

---

initialize

---

Initialize the MH-within-Gibbs algorithm for DDT-LCM

---

### Description

Initialize the MH-within-Gibbs algorithm for DDT-LCM

### Usage

```
initialize(
  K,
  data,
  item_membership_list,
  c = 1,
  c_order = 1,
  method_lcm = "random",
  method_dist = "euclidean",
  method_hclust = "ward.D",
  method_add_root = "min_cor",
  fixed_initials = list(),
  fixed_priors = list(),
  alpha = 0,
  theta = 0,
  ...
)
```

### Arguments

K	number of classes (integer)
data	an $N \times J$ matrix of multivariate binary responses, where $N$ is the number of individuals, and $J$ is the number of granular items
item_membership_list	a list of $G$ elements, where the $g$ -th element contains the column indices of data corresponding to items in major group $g$
c	hyperparameter of divergence function $a(t)$
c_order	equals 1 (default) or 2 to choose divergence function $a(t) = c/(1-t)$ or $c/(1-t)^2$ .
method_lcm	a character. If random (default), the initial LCM parameters will be random values. If poLCA, the initial LCM parameters will be EM algorithm estimates from the poLCA function.
method_dist	string specifying the distance measure to be used in <code>dist()</code> . This must be one of "euclidean" (defaults), "maximum", "manhattan", "canberra", "binary" or "minkowski". Any unambiguous substring can be given.
method_hclust	string specifying the distance measure to be used in <code>hclust()</code> . This should be (an unambiguous abbreviation of) one of "ward.D" (defaults), "ward.D2", "single", "complete", "average" (= UPGMA), "mcquitty" (= WPGMA), "median" (= WPGMC) or "centroid" (= UPGMC).



<code>method_add_root</code>	string specifying the method to add the initial branch to the tree output from <code>hclust()</code> . This should be one of "min_cor" (the absolute value of the minimum between-class correlation; default) or "sample_ddt" (randomly sample a small divergence time from the DDT process with $c = 100$ )
<code>fixed_initials</code>	a named list of fixed initial values, including the initial values for tree ("phylo4d"), <code>response_prob</code> , <code>class_probability</code> , <code>class_assignments</code> , <code>Sigma_by_group</code> , and <code>c</code> . Default is NULL. See
<code>fixed_priors</code>	a named list of fixed prior hyperparameters, including the the Gamma prior for <code>c</code> , inverse-Gamma prior for <code>sigma_g^2</code> , and Dirichlet prior for <code>pi</code> . Moreover, we allow for a type III generalized logistic distribution such that $f(\eta; a_{pg}) = \theta$ . This becomes a standard logistic distribution when $a_{pg} = 1$ . See Dalla Valle, L., Leisen, F., Rossini, L., & Zhu, W. (2021). A Pólya–Gamma sampler for a generalized logistic regression. <i>Journal of Statistical Computation and Simulation</i> , 91(14), 2899-2916. An example input list is <code>list("shape_c" = 1, "rate_c" = 1, "shape_sigma" = rep(2, G), "rate_sigma" = rep(2, G), "a_pg" = 1.0)</code> , where <code>G</code> is the number of major item groups. Default is NULL.
<code>alpha, theta</code>	hyperparameter of branching probability $a(t) \text{Gamma}(m-\alpha) / \text{Gamma}(m+1+\theta)$ For DDT, $\alpha = \theta = 0$
<code>...</code>	optional arguments for the <code>poLCA</code> function

**Value**

phylo4d object of tree topology

**See Also**

[ddtlcm\\_fit\(\)](#)

Other initialization functions: [initialize\\_hclust\(\)](#), [initialize\\_poLCA\(\)](#)

**Examples**

```
# load the MAP tree structure obtained from the real HCHS/SOL data
data(data_synthetic)
# extract elements into the global environment
list2env(setNames(data_synthetic, names(data_synthetic)), envir = globalenv())
K <- 3
G <- length(item_membership_list)
fixed_initials <- list("c" = 5)
fixed_priors <- list("rate_sigma" = rep(3, G), "shape_c" = 2, "rate_c" = 2)
initials <- initialize(K, data = response_matrix, item_membership_list,
  c=1, c_order=1, fixed_initials = fixed_initials, fixed_priors = fixed_priors)
```

---

initialize\_hclust      *Estimate an initial binary tree on latent classes using hclust()*

---

### Description

Estimate an initial binary tree on latent classes using hclust()

### Usage

```
initialize_hclust(
  leaf_data,
  c,
  c_order = 1,
  method_dist = "euclidean",
  method_hclust = "ward.D",
  method_add_root = "min_cor",
  alpha = 0,
  theta = 0,
  ...
)
```

### Arguments

leaf_data	a K by J matrix of $\text{logit}(\theta_{kj})$
c	hyperparameter of divergence function $a(t)$
c_order	equals 1 (default) or 2 to choose divergence function
method_dist	string specifying the distance measure to be used in dist(). This must be one of "euclidean", "maximum", "manhattan", "canberra", "binary" or "minkowski". Any unambiguous substring can be given.
method_hclust	string specifying the distance measure to be used in hclust(). This should be (an unambiguous abbreviation of) one of "ward.D", "ward.D2", "single", "complete", "average" (= UPGMA), "mcquitty" (= WPGMA), "median" (= WPGMC) or "centroid" (= UPGMC).
method_add_root	string specifying the method to add the initial branch to the tree output from hclust(). This should be one of "min_cor" (the absolute value of the minimum between-class correlation) or "sample_ddt" (randomly sample a small divergence time from the DDT process with a large $c = 100$ )
alpha, theta	hyperparameter of branching probability $a(t) \Gamma(m-\alpha) / \Gamma(m+1+\theta)$ For DDT, $\alpha = \theta = 0$
...	optional arguments for the poLCA function

### Value

phylo4d object of tree topology

**See Also**

Other initialization functions: [initialize\(\)](#), [initialize\\_poLCA\(\)](#)

---

initialize_poLCA	<i>Estimate an initial response profile from latent class model using poLCA()</i>
------------------	---

---

**Description**

Estimate an initial response profile from latent class model using poLCA()

**Usage**

```
initialize_poLCA(K, data, ...)
```

**Arguments**

K	number of latent classes
data	a N by J observed binary matrix, where the i,j-th element is the response of item j for individual i
...	optional arguments for the poLCA function

**Value**

a K by J probability matrix, the k,j-th entry being the response probability to item j of an individual in class k

**See Also**

Other initialization functions: [initialize\(\)](#), [initialize\\_hclust\(\)](#)

---

initialize_randomLCM	<i>Provide a random initial response profile based on latent class mode</i>
----------------------	---

---

**Description**

Provide a random initial response profile based on latent class mode

**Usage**

```
initialize_randomLCM(K, data)
```

**Arguments**

K	number of latent classes
data	a N by J observed binary matrix, where the i,j-th element is the response of item j for individual i

**Value**

a K by J probability matrix, the k,j-th entry being the response probability to item j of an individual in class k

---

J_n	<i>Compute factor in the exponent of the divergence time distribution</i>
-----	---

---

**Description**

Compute factor in the exponent of the divergence time distribution

**Usage**

J\_n(l, r)

**Arguments**

l	number of data points to the left
r	number of data points to the right

---

logit	<i>The logistic function</i>
-------	------------------------------

---

**Description**

The logit function:  $f(x) = \log(x / (1/x))$ . Large absolute values of x will be truncated to +/- 5 after logit transformation according to its sign.

**Usage**

logit(x)

**Arguments**

x	a value or a numeric vector between 0 and 1 (exclusive)
---	---

**Value**

a number or real-valued vector

**Examples**

```
logit(0.2)
logit(c(0.2, 0.6, 0.95))
```

---

logllk_ddt	<i>Calculate loglikelihood of a DDT, including the tree structure and node parameters</i>
------------	---

---

**Description**

Calculate loglikelihood of a DDT, including the tree structure and node parameters

**Usage**

```
logllk_ddt(
  c,
  c_order,
  Sigma_by_group,
  tree_phylo4d,
  item_membership_list,
  tree_structure_old = NULL,
  dist_mat_old = NULL
)
```

**Arguments**

<code>c</code>	a positive number for the divergence hyperparameter. A larger value implies earlier divergence on the tree
<code>c_order</code>	equals 1 if using divergence function $a(t) = c / (1-t)$ , or 2 if $a(t) = c / (1 - t)^2$ . Default is 1
<code>Sigma_by_group</code>	a vector of diffusion variances of G groups from the previous iteration
<code>tree_phylo4d</code>	a "phylo4d" object
<code>item_membership_list</code>	a list of G elements, where the g-th element contains the column indices of data corresponding to items in major group g
<code>tree_structure_old</code>	a list of at least named elements: loglikelihoods of the input tree topology and divergence times. These can be directly obtained from the return of this function. Default is NULL. If given a list, then computation of the loglikelihoods will be skipped to save time. This is useful in the Metropolis-Hasting algorithm when the previous proposal is not accepted.
<code>dist_mat_old</code>	a tree-structured covariance matrix from a given tree. Default is NULL.

**Value**

a numeric of loglikelihood

**See Also**

Other likelihood functions: [logllk\\_ddt\\_lcm\(\)](#), [logllk\\_div\\_time\\_one\(\)](#), [logllk\\_div\\_time\\_two\(\)](#), [logllk\\_lcm\(\)](#), [logllk\\_location\(\)](#), [logllk\\_tree\\_topology\(\)](#)

---

<code>logllk_ddt_lcm</code>	<i>Calculate loglikelihood of the DDT-LCM</i>
-----------------------------	---

---

**Description**

Calculate loglikelihood of the DDT-LCM

**Usage**

```
logllk_ddt_lcm(
  c,
  Sigma_by_group,
  tree_phylo4d,
  item_membership_list,
  tree_structure_old = NULL,
  dist_mat_old = NULL,
  response_matrix,
  leaf_data,
  prior_class_probability,
  prior_dirichlet,
  ClassItem,
  Class_count
)
```

**Arguments**

`c` a positive number for the divergence hyperparameter. A larger value implies earlier divergence on the tree

`Sigma_by_group` a vector of diffusion variances of G groups

`tree_phylo4d` a "phylo4d" object

`item_membership_list` a list of G elements, where the g-th element contains the column indices of data corresponding to items in major group g

`tree_structure_old` a list of at least named elements: loglikelihoods of the input tree topology and divergence times. These can be directly obtained from the return of this function. Default is NULL. If given a list, then computation of the loglikelihoods will be skipped to save time. This is useful in the Metropolis-Hasting algorithm when the previous proposal is not accepted.

`dist_mat_old` a tree-structured covariance matrix from a given tree. Default is NULL.

response_matrix	a N by J binary matrix, where the i,j-th element is the response of item j for individual i
leaf_data	a K by J matrix of $\text{logit}(\theta_{kj})$
prior_class_probability	a length K vector, where the k-th element is the probability of assigning an individual to class k. It does not have to sum up to 1
prior_dirichlet	a vector of length K. The Dirichlet prior of class probabilities
ClassItem	a K by J matrix, where the k,j-th element counts the number of individuals that belong to class k have a positive response to item j
Class_count	a length K vector, where the k-th element counts the number of individuals belonging to class k

**Value**

a numeric of loglikelihood

**See Also**

Other likelihood functions: [logllk\\_ddt\(\)](#), [logllk\\_div\\_time\\_one\(\)](#), [logllk\\_div\\_time\\_two\(\)](#), [logllk\\_lcm\(\)](#), [logllk\\_location\(\)](#), [logllk\\_tree\\_topology\(\)](#)

---

logllk\_div\_time\_one    *Compute loglikelihood of divergence times for  $a(t) = c/(1-t)$*

---

**Description**

Compute loglikelihood of divergence times for  $a(t) = c/(1-t)$

**Usage**

```
logllk_div_time_one(c, l, r, t)
```

**Arguments**

c	a positive number for the divergence hyperparameter. A larger value implies earlier divergence on the tree
l	number of data points to the left
r	number of data points to the right
t	a number in the interval (0, 1) indicating the divergence time

**See Also**

Other likelihood functions: [logllk\\_ddt\(\)](#), [logllk\\_ddt\\_lcm\(\)](#), [logllk\\_div\\_time\\_two\(\)](#), [logllk\\_lcm\(\)](#), [logllk\\_location\(\)](#), [logllk\\_tree\\_topology\(\)](#)

---

logllk\_div\_time\_two     *Compute loglikelihood of divergence times for  $a(t) = c/(1-t)^2$*

---

### Description

Compute loglikelihood of divergence times for  $a(t) = c/(1-t)^2$

### Usage

```
logllk_div_time_two(c, l, r, t)
```

### Arguments

c	a positive number for the divergence hyperparameter. A larger value implies earlier divergence on the tree
l	number of data points to the left
r	number of data points to the right
t	a number in the interval (0, 1) indicating the divergence time

### See Also

Other likelihood functions: [logllk\\_ddt\(\)](#), [logllk\\_ddt\\_lcm\(\)](#), [logllk\\_div\\_time\\_one\(\)](#), [logllk\\_lcm\(\)](#), [logllk\\_location\(\)](#), [logllk\\_tree\\_topology\(\)](#)

---

logllk\_lcm     *Calculate loglikelihood of the latent class model, conditional on tree structure*

---

### Description

Calculate loglikelihood of the latent class model, conditional on tree structure

### Usage

```
logllk_lcm(
  response_matrix,
  leaf_data,
  prior_class_probability,
  prior_dirichlet,
  ClassItem,
  Class_count
)
```



**Arguments**

response_matrix	a N by J binary matrix, where the i,j-th element is the response of item j for individual i
leaf_data	a K by J matrix of $\text{logit}(\theta_{kj})$
prior_class_probability	a length K vector, where the k-th element is the probability of assigning an individual to class k. It does not have to sum up to 1
prior_dirichlet	a vector of length K. The Dirichlet prior of class probabilities
ClassItem	a K by J matrix, where the k,j-th element counts the number of individuals that belong to class k have a positive response to item j
Class_count	a length K vector, where the k-th element counts the number of individuals belonging to class k

**Value**

a numeric of loglikelihood

**See Also**

Other likelihood functions: [logllk\\_ddt\(\)](#), [logllk\\_ddt\\_lcm\(\)](#), [logllk\\_div\\_time\\_one\(\)](#), [logllk\\_div\\_time\\_two\(\)](#), [logllk\\_location\(\)](#), [logllk\\_tree\\_topology\(\)](#)

---

logllk_location	<i>Compute log likelihood of parameters</i>
-----------------	---

---

**Description**

Compute the marginal log likelihood of the parameters on the leaves of a tree

**Usage**

```
logllk_location(
  tree_phylo4d,
  Sigma_by_group,
  item_membership_list,
  dist_mat = NULL,
  tol = 1e-07
)
```

**Arguments**

tree_phylo4d	a "phylo4d" object
Sigma_by_group	a vector of diffusion variances of G groups
item_membership_list	a list of G elements, where the g-th element contains the column indices of data corresponding to items in major group g
dist_mat	a tree-structured covariance matrix from a given tree. Default is NULL. If given a matrix, then computation of the covariance matrix will be skipped to save time. This is useful in the Metropolis-Hasting algorithm when the previous proposal is not accepted.
tol	a small number to prevent underflow when computing eigenvalues

**Value**

A list of two elements: a numeric loglikelihood, a covariance matrix of the input tree

**See Also**

Other likelihood functions: [logllk\\_ddt\(\)](#), [logllk\\_ddt\\_lcm\(\)](#), [logllk\\_div\\_time\\_one\(\)](#), [logllk\\_div\\_time\\_two\(\)](#), [logllk\\_lcm\(\)](#), [logllk\\_tree\\_topology\(\)](#)

---

logllk\_tree\_topology    *Compute loglikelihood of the tree topology*

---

**Description**

Compute loglikelihood of the tree topology

**Usage**

```
logllk_tree_topology(l, r)
```

**Arguments**

l	number of data points to the left
r	number of data points to the right

**See Also**

Other likelihood functions: [logllk\\_ddt\(\)](#), [logllk\\_ddt\\_lcm\(\)](#), [logllk\\_div\\_time\\_one\(\)](#), [logllk\\_div\\_time\\_two\(\)](#), [logllk\\_lcm\(\)](#), [logllk\\_location\(\)](#)

---

log_expit	<i>Numerically accurately compute <math>f(x) = \log(x / (1/x))</math>.</i>
-----------	--

---

**Description**

Numerically accurately compute  $f(x) = \log(x / (1/x))$ .

**Usage**

```
log_expit(x)
```

**Arguments**

x a value or a numeric vector between 0 and 1 (exclusive)

**Value**

a number or real-valued vector

---

parameter_diet	<i>Parameters for the HCHS dietary recall data example</i>
----------------	--

---

**Description**

A list of five variables containing the food items and parameter estimates obtained from MCMC chains. The real multivariate binary dataset is not included for privacy. The variables are as follows:

**Usage**

```
data(parameter_diet)
```

**Format**

An object of class `list` of length 5.

**Details**

- `item_membership_list`. A list of  $G = 7$  elements, where the  $g$ -th element contains the indices of items belonging to major food group  $g$ .
- `item_name_list`. A list of  $G = 7$  elements, where the  $g$ -th element contains the item labels of items in major food group  $g$ , and the name of the  $g$ -th element is the major food group label.
- `tree_phylo`. The maximum a posterior tree estimate with  $K = 6$  leaves obtained from the real HCHS data. Class "phylo".
- `class_probability`. A  $K$ -vector with entries between 0 and 1. The posterior mean estimate for class probabilities obtained from the real HCHS data.
- `Sigma_by_group`. A  $G$ -vector greater than 0. The posterior mean estimate for group-specific diffusion variances obtained from the real HCHS data.

**References**

<https://arxiv.org/abs/2306.04700>

---

plot.ddt_lcm	<i>Create trace plots of DDT-LCM parameters</i>
--------------	---

---

**Description**

Create trace plots of DDT-LCM parameters

**Usage**

```
## S3 method for class 'ddt_lcm'
plot(
  x,
  parameter_names = c("responseprob_1,1,1", "classprob_1", "c", "diffusionvar_1"),
  burnin = 50,
  ...
)
```

**Arguments**

x	a "ddt_lcm" object
parameter_names	a character vector to specify the parameters to be plotted. Each element can take the be 1) of format "parameter_index" to plot specific parameters with specific indices, 2) of format "parameter" to plot the parameters across all indices, or 3) equal to "all" to plot all parameters in the model. For 1), the item response probabilities should be named "responseprob_class,group,item"; the class probabilities should be named "classprob_class"; the divergence function parameter is "c"; the group-specific diffusion variances should be named "diffusionvar_group". For 2), "responseprob" to plot all item response probabilities; "classprob" to plot all class probabilities; "diffusionvar" to plot all diffusion variances.
burnin	the number of posterior samples as burn-in, which will not be plotted.
...	Further arguments passed to each method

**Value**

NULLs

**Examples**

```

data(result_diet_1000iters)
# Plot "c" for the divergence function parameter; "diffusionvar_1" for diffusion variance of group 1
plot(x = result_diet_1000iters, parameter_names = c("c", "diffusionvar_1"), burnin = 500)
# Plot "responseprob_1,1,1" for the class 1 response probability of item 3 in major group 2
plot(x = result_diet_1000iters, parameter_names = "responseprob_1,1,1", burnin = 500)
# Plot "classprob_1" for the probability of being assigned to class 1
plot(x = result_diet_1000iters, parameter_names = "classprob_1", burnin = 500)
# plot all class probabilities
plot(x = result_diet_1000iters, parameter_names = "classprob", burnin = 500)
# plot all diffusion variances
plot(x = result_diet_1000iters, "diffusionvar", burnin = 500)

```

---

```
plot.summary.ddt_lcm
```

*Plot the MAP tree and class profiles of summarized DDT-LCM results*

---

**Description**

Plot the MAP tree and class profiles of summarized DDT-LCM results

**Usage**

```

## S3 method for class 'summary.ddt_lcm'
plot(
  x,
  log = TRUE,
  plot_option = c("all", "profile", "tree"),
  item_name_list = NULL,
  color_palette = c("#E69F00", "#56B4E9", "#009E73", "#000000", "#0072B2", "#D55E00",
    "#CC79A7", "#F0E442", "#999999"),
  ...
)

```

**Arguments**

<code>x</code>	a "summary.ddt_lcm" object
<code>log</code>	Default argument passed to plot(). Not used.
<code>plot_option</code>	option to select which part of the plot to return. If "all", return the plot of MAP tree on the left and the plot of class profiles on the right. If "profile", only return the plot of class profiles. If "tree", only return the plot of MAP tree.
<code>item_name_list</code>	a named list of G elements, where the g-th element contains a vector of item names for items in <code>item_membership_list[[g]]</code> . The name of the g-th element is the name of the major item group.
<code>color_palette</code>	a vector of color names. Default is a color-blinded friendly palette.
<code>...</code>	Further arguments passed to each method

**Value**

a ggplot2 object. If plot\_option is "all", then a plot with maximum a posterior tree structure on the left and a bar plot of item response probabilities (with 95% credible intervals and class probabilities) on the right. If plot\_option is "profile", then only a bar plot of item response probabilities. If plot\_option is "tree", then only a plot of the tree structure.

**Examples**

```
data(result_diet_1000iters)
burnin <- 500
summarized_result <- summary(result_diet_1000iters, burnin, relabel = TRUE, be_quiet = TRUE)
plot(x = summarized_result, item_name_list = NULL, plot_option = "all")
```

---

```
plot_tree_with_barplot
```

*Plot the MAP tree and class profiles (bar plot) of summarized DDT-LCM results*

---

**Description**

Plot the MAP tree and class profiles (bar plot) of summarized DDT-LCM results

**Usage**

```
plot_tree_with_barplot(
  tree_with_parameter,
  response_prob,
  item_membership_list,
  item_name_list = NULL,
  class_probability = NULL,
  class_probability_lower = NULL,
  class_probability_higher = NULL,
  color_palette = c("#E69F00", "#56B4E9", "#009E73", "#000000", "#0072B2", "#D55E00",
    "#CC79A7", "#F0E442", "#999999"),
  return_separate_plots = FALSE
)
```

**Arguments**

`tree_with_parameter` a "phylo4d" tree with node parameters

`response_prob` a K by J matrix, where the k,j-th element is the response probability of item j for individuals in class k

`item_membership_list` a list of G elements, where the g-th element contains the column indices of the observed data matrix corresponding to items in major group g

`item_name_list` a named list of  $G$  elements, where the  $g$ -th element contains a vector of item names for items in `item_membership_list[[g]]`. The name of the  $g$ -th element is the name of the major item group.

`class_probability` a length  $K$  vector, where the  $k$ -th element is the probability of assigning an individual to class  $k$ . It does not have to sum up to 1

`class_probability_lower` a length  $K$  vector, 2.5% quantile of posterior the distribution.

`class_probability_higher` a length  $K$  vector, 97.5% quantile of posterior the distribution.

`color_palette` a vector of color names. Default is a color-blinded friendly palette.

`return_separate_plots` If FALSE (default), print the combined plot of MAP tree and class profiles. If TRUE, return the tree plot, class profile plot, and `data.table` used to create the plots in a list, without printing the combined plot.

**Value**

a `ggplot2` object. A bar plot of item response probabilities.

**Examples**

```
# load the MAP tree structure obtained from the real HCHS/SOL data
data(data_synthetic)
# extract elements into the global environment
list2env(setNames(data_synthetic, names(data_synthetic)), envir = globalenv())
plot_tree_with_barplot(tree_with_parameter, response_prob, item_membership_list)
```

---

`plot_tree_with_heatmap`

*Plot the MAP tree and class profiles (heatmap) of summarized DDT-LCM results*

---

**Description**

Plot the MAP tree and class profiles (heatmap) of summarized DDT-LCM results

**Usage**

```
plot_tree_with_heatmap(
  tree_with_parameter,
  response_prob,
  item_membership_list
)
```

**Arguments**

tree\_with\_parameter a "phylo4d" tree with node parameters

response\_prob a K by J matrix, where the k,j-th element is the response probability of item j for individuals in class k

item\_membership\_list a list of G elements, where the g-th element contains the column indices of the observed data matrix corresponding to items in major group g

**Value**

a ggplot2 object. A plot with the tree structure on the left and a heatmap of item response probabilities on the right, with indication of item group memberships beneath the heatmap.

**Examples**

```
# load the MAP tree structure obtained from the real HCHS/SOL data
data(data_synthetic)
# extract elements into the global environment
list2env(setNames(data_synthetic, names(data_synthetic)), envir = globalenv())
plot_tree_with_heatmap(tree_with_parameter, response_prob, item_membership_list)
```

---

predict.ddt_lcm	<i>Prediction of class memberships from posterior predictive distributions</i>
-----------------	--

---

**Description**

Predict individual class memberships based on posterior predictive distributions. For each posterior sample, let the class memberships be modal assignments. Then aggregate over all posterior samples to obtain the most likely assigned classes.

**Usage**

```
## S3 method for class 'ddt_lcm'
predict(object, data, burnin = 3000, ...)
```

**Arguments**

object a ddt\_lcm object

data an NxJ matrix of multivariate binary responses, where N is the number of individuals, and J is the number of granular items

burnin number of samples to discard from the posterior chain as burn-ins. Default is 3000.

... Further arguments passed to each method



**Value**

a list of the following named elements:

`class_assignments` an integer vector of individual predicted class memberships taking values in 1, ..., K

`predictive_probs` a N x K matrix of probabilities, where the (i,k)-th element is the probability that the i-th individual is predicted to belong to class k.

**Examples**

```
data(result_diet_1000iters)
burnin <- 500
predicted <- predict(result_diet_1000iters, result_diet_1000iters$data, burnin)
```

---

predict.summary.ddt\_lcm

*Prediction of class memberships from posterior summaries*

---

**Description**

Predict individual class memberships based on posterior summary (point estimates of model parameters). The predicted class memberships are modal assignments.

**Usage**

```
## S3 method for class 'summary.ddt_lcm'
predict(object, data, ...)
```

**Arguments**

<code>object</code>	a "summary.ddt_lcm" object
<code>data</code>	an NxJ matrix of multivariate binary responses, where N is the number of individuals, and J is the number of granular items
<code>...</code>	Further arguments passed to each method

**Value**

a list of the following named elements:

`class_assignments` an integer vector of individual predicted class memberships taking values in 1, ..., K

`predictive_probs` a N x K matrix of probabilities, where the (i,k)-th element is the probability that the i-th individual is predicted to belong to class k.

**Examples**

```
data(result_diet_1000iters)
burnin <- 500
summarized_result <- summary(result_diet_1000iters, burnin, relabel = TRUE, be_quiet = TRUE)
predicted <- predict(summarized_result, result_diet_1000iters$data)
```

---

`print.ddt_lcm`      *Print out setup of a ddt\_lcm model*

---

**Description**

Print out setup of a ddt\_lcm model

**Usage**

```
## S3 method for class 'ddt_lcm'
print(x, ...)
```

**Arguments**

`x`                    a "ddt\_lcm" object  
`...`                Further arguments passed to each method

**See Also**

Other ddt\_lcm results: [print.summary.ddt\\_lcm\(\)](#), [summary.ddt\\_lcm\(\)](#)

**Examples**

```
data(result_diet_1000iters)
print(result_diet_1000iters)
```

---

`print.summary.ddt_lcm`      *Print out summary of a ddt\_lcm model*

---

**Description**

Print out summary of a ddt\_lcm model

**Usage**

```
## S3 method for class 'summary.ddt_lcm'
print(x, digits = 3L, ...)
```

**Arguments**

x a "summary.ddt\_lcm" object  
 digits integer indicating the number of decimal places (round) to be used.  
 ... Further arguments passed to each method

**See Also**

Other ddt\_lcm results: [print.ddt\\_lcm\(\)](#), [summary.ddt\\_lcm\(\)](#)

**Examples**

```
data(result_diet_1000iters)
burnin <- 500
summarized_result <- summary(result_diet_1000iters, burnin, relabel = TRUE, be_quiet = TRUE)
print(summarized_result)
```

---

proposal\_log\_prob      *Calculate proposal likelihood*

---

**Description**

Given an old tree, propose a new tree and calculate the original and proposal tree likelihood in the DDT process

**Usage**

```
proposal_log_prob(
  old_tree_phylo4,
  tree_kept,
  old_detach_pa_div_time,
  old_pa_detach_node_label,
  old_detach_node_label,
  new_div_time,
  new_attach_root,
  new_attach_to,
  c,
  c_order = 1
)
```

**Arguments**

old\_tree\_phylo4 the old "phylo4" object  
 tree\_kept the remaining "phylo" tree after detachment  
 old\_detach\_pa\_div\_time a number in (0, 1) indicating the divergence time of the detached node on the old tree

old_pa_detach_node_label	a character label of the parent of the detached node on the old tree
old_detach_node_label	a character label of the detached node on the old tree
new_div_time	a number in (0, 1) indicating the divergence time at which the detached subtree will be re-attached on the proposal tree
new_attach_root, new_attach_to	a character label of the starting and ending nodes of the branch on the proposal tree, which the detached subtree will be re-attached to
c	hyparameter of divergence function $a(t)$
c_order	equals 1 (default) or 2 to choose divergence function

**Value**

a list containing the following elements:

q\_new a "phylo" tree detached from the input tree

q\_old the remaining "phylo" tree after detachment

---

quiet	<i>Suppress print from cat()</i>
-------	----------------------------------

---

**Description**

Suppress print from cat()

**Usage**

```
quiet(x, be_quiet = TRUE)
```

**Arguments**

x	evaluation of a statement that may explicitly or implicitly involve cat()
be_quiet	logical. TRUE to suppress print from cat(); FALSE to continue printing

---

random\_detach\_subtree *Metropolis-Hasting algorithm for sampling tree topology and branch lengths from the DDT branching process.*

---

**Description**

Randomly detach a subtree from a given tree

**Usage**

```
random_detach_subtree(tree_phylo4)
```

**Arguments**

tree\_phylo4 a "phylo4" object

**Value**

a list containing the following elements:

tree\_detached a "phylo" tree detached from the input tree

tree\_kept the remaining "phylo" tree after detachment

pa\_detach\_node\_label a character label of the parent of the node from which the detachment happens

pa\_div\_time a number in (0, 1) indicating the divergence time of the parent of the detached node

detach\_div\_time a number in (0, 1) indicating the divergence time of the detached node

detach\_node\_label a character label of the parent of the detached node

**See Also**

Other sample trees: [attach\\_subtree\(\)](#), [reattach\\_point\(\)](#)

**Examples**

```
library(phylobase)
# load the MAP tree structure obtained from the real HCHS/SOL data
data(data_synthetic)
# extract elements into the global environment
list2env(setNames(data_synthetic, names(data_synthetic)), envir = globalenv())
detachment <- random_detach_subtree(extractTree(tree_with_parameter))
```

---

reattach_point	<i>Attach a subtree to a given DDT at a randomly selected location</i>
----------------	--

---

**Description**

Attach a subtree to a given DDT at a randomly selected location

**Usage**

```
reattach_point(tree_kept, c, c_order = 1, theta = 0, alpha = 0)
```

**Arguments**

tree_kept	the tree to be attached to
c	hyparameter of divergence function $a(t)$
c_order	equals 1 (default) or 2 to choose divergence function $a(t) = c/(1-t)$ or $c/(1-t)^2$ .
alpha, theta	hyparameter of branching probability $a(t) \text{Gamma}(m-\alpha) / \text{Gamma}(m+1+\theta)$ For DDT, $\alpha = \theta = 0$ . For general multifurcating tree from a Pitman-Yor process, specify positive values to alpha and theta. It is, however, recommended using $\alpha = \theta = 0$ in inference because multifurcating trees have not been tested rigorously.

**Value**

a list of the following objects:

div\_time a numeric value of newly sampled divergence time. Between 0 and 1.

root\_node a character. Label of the root node of tree\_kept.

root\_child a character. Label of the child node of the root of tree\_kept.

div\_dist\_to\_root\_child a N-vector with integer entries from 1, ..., K. The initial values for individual class assignments.

**See Also**

Other sample trees: [attach\\_subtree\(\)](#), [random\\_detach\\_subtree\(\)](#)

---

result\_diet\_1000iters *Result of fitting DDT-LCM to a semi-synthetic data example*

---

### Description

This is a "ddtlcm" object obtained from running `ddtlcm_fit` to a semi-synthetic dataset with 1000 posterior samples (for the sake of time). See [ddtlcm\\_fit](#) for description of the object.

### Usage

```
data(result_diet_1000iters)
```

### Format

A list with 8 elements

---

sample\_class\_assignment

*Sample individual class assignments  $Z_i, i = 1, \dots, N$*

---

### Description

Sample individual class assignments  $Z_i, i = 1, \dots, N$

### Usage

```
sample_class_assignment(
  data,
  leaf_data,
  a_pg,
  auxiliary_mat,
  class_probability
)
```

### Arguments

<code>data</code>	a N by J binary matrix, where the i,j-th element is the response of item j for individual i
<code>leaf_data</code>	a K by J matrix of $\text{logit}(\theta_{kj})$
<code>a_pg</code>	a N by J matrix of hyperparameters of the generalized logistic distribution
<code>auxiliary_mat</code>	a N by J matrix of truncated normal variables from previous iteration
<code>class_probability</code>	a length K vector, where the k-th element is the probability of assigning an individual to class k. It does not have to sum up to 1

**Value**

a vector of length N, where the i-th element is the class assignment of individual i

---

sample_c_one	<i>Sample divergence function parameter c for <math>a(t) = c / (1-t)</math> through Gibbs sampler</i>
--------------	---

---

**Description**

Sample divergence function parameter c for  $a(t) = c / (1-t)$  through Gibbs sampler

**Usage**

```
sample_c_one(shape0, rate0, tree_structure)
```

**Arguments**

shape0	shape of the Inverse-Gamma prior
rate0	rate of the Inverse-Gamma prior
tree_structure	a data.frame containing the divergence times and number of data points to the left and right branches of internal nodes on the tree

**Value**

a numeric value of the newly sampled c

---

sample_c_two	<i>Sample divergence function parameter c for <math>a(t) = c / (1-t)^2</math> through Gibbs sampler</i>
--------------	---

---

**Description**

Sample divergence function parameter c for  $a(t) = c / (1-t)^2$  through Gibbs sampler

**Usage**

```
sample_c_two(shape0, rate0, tree_structure)
```

**Arguments**

shape0	shape of the Inverse-Gamma prior
rate0	rate of the Inverse-Gamma prior
tree_structure	a data.frame containing the divergence times and number of data points to the left and right branches of internal nodes on the tree



**Value**

a numeric value of the newly sampled  $c$

---

sample\_leaf\_locations\_pg

*Sample the leaf locations and Polya-Gamma auxilliary variables*

---

**Description**

Sample the leaf locations and Polya-Gamma auxilliary variables

**Usage**

```
sample_leaf_locations_pg(
  item_membership_list,
  dist_mat_old,
  Sigma_by_group,
  pg_mat,
  a_pg,
  auxiliary_mat,
  auxiliary_mat_range,
  class_assignments
)
```

**Arguments**

`item_membership_list` a vector of  $G$  elements, each indicating the number of items in this group

`dist_mat_old` a list of leaf covariance matrix from the previous iteration. The list has length  $G$ , the number of item groups

`Sigma_by_group` a vector of length  $G$ , each denoting the variance of the brownian motion

`pg_mat` a  $K$  by  $J$  matrix of PG variables from the previous iteration

`a_pg` a  $N$  by  $J$  matrix of hyperparameters of the generalized logistic distribution

`auxiliary_mat` a  $N$  by  $J$  matrix of truncated normal variables from previous iteration

`auxiliary_mat_range` a list of two named elements: `lb` and `ub`. Each is an  $N$  by  $J$  matrix of the lower/upper bounds of the truncated normal variables.

`class_assignments` an integer vector of length  $N$  for the individual class assignments. Each element takes value in  $1, \dots, K$ .

**Value**

a named list of three matrices: the newly sampled leaf parameters, the Polya-gamma random variables, and the auxiliary truncated normal variables

---

sample_sigmasq	<i>Sample item group-specific variances through Gibbs sampler</i>
----------------	---

---

**Description**

Sample item group-specific variances through Gibbs sampler

**Usage**

```
sample_sigmasq(shape0, rate0, dist_mat, item_membership_list, locations)
```

**Arguments**

shape0	a vector of G elements, each being the shape of the Inverse-Gamma prior of group g
rate0	a vector of G elements, each being the rate of the Inverse-Gamma prior of group g
dist_mat	a list, containing the KxK tree-structured matrix of leaf nodes, where K is the number of leaves / latent classes, and SVD components
item_membership_list	a vector of G elements, each indicating the number of items in this group
locations	a KxJ matrix of leaf parameters

**Value**

a numeric vector of G elements, each being the newly sampled variance of the latent location of this group

---

sample_tree_topology	<i>Sample a new tree topology using Metropolis-Hastings through randomly detaching and re-attaching subtrees</i>
----------------------	--

---

**Description**

Sample a new tree topology using Metropolis-Hastings through randomly detaching and re-attaching subtrees

**Usage**

```
sample_tree_topology(
  tree_phylo4d_old,
  Sigma_by_group,
  item_membership_list,
  c,
  c_order = 1,
  tree_structure_old = NULL,
  dist_mat_old = NULL
)
```

**Arguments**

`tree_phylo4d_old` a phylo4d object of tree from the previous iteration

`Sigma_by_group` a vector of diffusion variances of G groups from the previous iteration

`item_membership_list` a vector of G elements, each indicating the number of items in this group

`c` hyparameter of divergence function  $a(t)$

`c_order` equals 1 (default) or 2 to choose divergence function

`tree_structure_old` a data.frame of tree structure from the previous iteration. Each row contains information of an internal node, including divergence times, number of data points traveling through the left and right branches

`dist_mat_old` a list of leaf covariance matrix from the previous iteration. The list has length G, the number of item groups

**Value**

a numeric vector of G elements, each being the newly sampled variance of the latent location of this group

---

<code>simulate_DDT_tree</code>	<i>Simulate a tree from a DDT process. Only the tree topology and branch lengths are simulated, without node parameters.</i>
--------------------------------	--

---

**Description**

Simulate a tree from a DDT process. Only the tree topology and branch lengths are simulated, without node parameters.

**Usage**

```
simulate_DDT_tree(K, c, c_order = 1, alpha = 0, theta = 0)
```

**Arguments**

K	number of leaves (classes) on the tree
c	hyparameter of divergence function $a(t)$
c_order	equals 1 (default) or 2 to choose divergence function $a(t) = c/(1-t)$ or $c/(1-t)^2$ .
alpha, theta	hyparameter of branching probability $a(t) \text{Gamma}(m-\alpha) / \text{Gamma}(m+1+\theta)$ For DDT, $\alpha = \theta = 0$ . For general multifurcating tree from a Pitman-Yor process, specify positive values to alpha and theta. It is, however, recommended using $\alpha = \theta = 0$ in inference because multifurcating trees have not been tested rigorously.

**Value**

A class "phylo" tree with K leaves. The leaf nodes are labeled "v1", ..., "vK", root node "u1", and internal nodes "u2", ..., "uK". Note that this tree does not contain any node parameters.

**References**

Knowles, D. A., & Ghahramani, Z. (2014). Pitman yor diffusion trees for bayesian hierarchical clustering. *IEEE transactions on pattern analysis and machine intelligence*, 37(2), 271-289.

**See Also**

Other simulate DDT-LCM data: [simulate\\_lcm\\_given\\_tree\(\)](#), [simulate\\_lcm\\_response\(\)](#), [simulate\\_parameter\\_on\\_tr](#)

**Examples**

```
K <- 6
c <- 5
c_order <- 1
tree1 <- simulate_DDT_tree(K, c, c_order)
tree2 <- simulate_DDT_tree(K, c, c_order, alpha = 0.4, theta = 0.1)
tree3 <- simulate_DDT_tree(K, c, c_order, alpha = 0.8, theta = 0.1)
```

---

```
simulate_lcm_given_tree
```

*Simulate multivariate binary responses from a latent class model given a tree*

---

**Description**

Generate multivariate binary responses from the following process: For individual  $i = 1, \dots, N$ ,  $Z_i \text{Categorical}_K(\text{prior\_class\_probability})$  For item  $j = 1, \dots, J$ ,  $Y_{ij} | Z_i = k \text{Binomial}(\text{class\_item\_probability}_{kj})$

**Usage**

```
simulate_lcm_given_tree(
  tree_phylo,
  N,
  class_probability = 1,
  item_membership_list,
  Sigma_by_group = NULL,
  root_node_location = 0,
  seed_parameter = 1,
  seed_response = 1
)
```

**Arguments**

`tree_phylo` a "phylo" tree with K leaves

`N` number of individuals

`class_probability` a length K vector, where the k-th element is the probability of assigning an individual to class k. It does not have to sum up to 1

`item_membership_list` a list of G elements, where the g-th element contains the column indices of the observed data matrix corresponding to items in major group g

`Sigma_by_group` a length-G vector for the posterior mean group-specific diffusion variances.

`root_node_location` the coordinate of the root node parameter. By default, the node parameter initiates at the origin so takes value 0. If a value, then the value will be repeated into a length J vector. If a vector, it must be of length J.

`seed_parameter` an integer random seed to generate parameters given the tree

`seed_response` an integer random seed to generate multivariate binary observations from LCM

**Value**

a named list of the following elements:

`tree_with_parameter` a "phylo4d" tree with K leaves.

`response_prob` a K by J matrix, where the k,j-th element is the response probability of item j for individuals in class k

`response_matrix` a K by J matrix with entries between 0 and 1 for the item response probabilities.

`class_probability` a K-vector with entries between 0 and 1 for the class probabilities. Entries should be nonzero and sum up to 1, or otherwise will be normalized

`class_assignments` a N-vector with integer entries from 1, ..., K. The initial values for individual class assignments.

`Sigma_by_group` a G-vector greater than 0. The initial values for the group-specific diffusion variances.

`c` a value greater than 0. The initial values for the group-specific diffusion variances.

`item_membership_list` same as input

**See Also**

Other simulate DDT-LCM data: [simulate\\_DDT\\_tree\(\)](#), [simulate\\_lcm\\_response\(\)](#), [simulate\\_parameter\\_on\\_tree\(\)](#)

**Examples**

```
# load the MAP tree structure obtained from the real HCHS/SOL data
data(parameter_diet)
# unlist the elements into variables in the global environment
list2env(setNames(parameter_diet, names(parameter_diet)), envir = globalenv())
# number of individuals
N <- 496
# set random seed to generate node parameters given the tree
seed_parameter = 1
# set random seed to generate multivariate binary observations from LCM
seed_response = 1
# simulate data given the parameters
sim_data <- simulate_lcm_given_tree(tree_phylo, N,
                                   class_probability, item_membership_list, Sigma_by_group,
                                   root_node_location = 0, seed_parameter = 1, seed_response = 1)
```

---

simulate\_lcm\_response *Simulate multivariate binary responses from a latent class model*

---

**Description**

Generate multivariate binary responses from the following process: For individual  $i = 1, \dots, N$ , draw  $Z_i$  from Categorical distribution with prior class probability (length  $K$ ). For item  $j = 1, \dots, J$ , given  $Z_i = k$ , draw  $Y_{ij}$  from Binomial with class-item probability

**Usage**

```
simulate_lcm_response(N, response_prob, class_probability)
```

**Arguments**

**N** number of individuals

**response\_prob** a  $K$  by  $J$  matrix, where the  $k,j$ -th element is the response probability of item  $j$  for individuals in class  $k$

**class\_probability** a length  $K$  vector, where the  $k$ -th element is the probability of assigning an individual to class  $k$ . It does not have to sum up to 1

**Value**

a named list of the following elements:

**response\_matrix** a  $K$  by  $J$  matrix with entries between 0 and 1 for the item response probabilities.

**class\_probability** a  $K$ -vector with entries between 0 and 1 for the class probabilities. Entries should be nonzero and sum up to 1, or otherwise will be normalized

**See Also**

Other simulate DDT-LCM data: [simulate\\_DDT\\_tree\(\)](#), [simulate\\_lcm\\_given\\_tree\(\)](#), [simulate\\_parameter\\_on\\_tree\(\)](#)

**Examples**

```
# number of latent classes
K <- 6
# number of items
J <- 78
response_prob <- matrix(runif(K*J), nrow = K)
class_probability <- rep(1/K, K)
# number of individuals
N <- 100
response_matrix <- simulate_lcm_response(N, response_prob, class_probability)
```

---

```
simulate_parameter_on_tree
```

*Simulate node parameters along a given tree.*

---

**Description**

Simulate node parameters along a given tree.

**Usage**

```
simulate_parameter_on_tree(
  tree_phylo,
  Sigma_by_group,
  item_membership_list,
  root_node_location = 0
)
```

**Arguments**

`tree_phylo` a "phylo" object containing the tree topology and branch lengths

`Sigma_by_group` a G-vector greater than 0. The initial values for the group-specific diffusion variances

`item_membership_list` a list of G elements, where the g-th element contains the indices of items in major group g

`root_node_location` the coordinate of the root node parameter. By default, the node parameter initiates at the origin so takes value 0. If a value, then the value will be repeated into a length J vector. If a vector, it must be of length J.

**Value**

A class "phylo4d" tree with K leaves with node parameters. The leaf nodes are labeled "v1", ..., "vK", root node "u1", and internal nodes "u2", ..., "uK".

**See Also**

Other simulate DDT-LCM data: [simulate\\_DDT\\_tree\(\)](#), [simulate\\_lcm\\_given\\_tree\(\)](#), [simulate\\_lcm\\_response\(\)](#)

**Examples**

```
library(ape)
tr_txt <- "(((v1:0.25, v2:0.25):0.65, v3:0.9):0.1);"
tree <- read.tree(text = tr_txt)
tree$node.label <- paste0("u", 1:Nnode(tree))
plot(tree, show.node.label = TRUE)
# create a list of item membership indices of 7 major groups
item_membership_list <- list()
num_items_per_group <- c(rep(10, 5), 15, 15)
G <- length(num_items_per_group)
j <- 0
for (g in 1:G) {
  item_membership_list[[g]] <- (j+1):(j+num_items_per_group[g])
  j <- j+num_items_per_group[g]
}
# variance of logit response probabilities of items in each group
Sigma_by_group <- c(rep(0.6**2, 5), rep(2**2, 2)) #rep(1**2, G)
set.seed(50)
tree_with_parameter <- simulate_parameter_on_tree(tree, Sigma_by_group, item_membership_list)
```

---

summary.ddt\_lcm

*Summarize the output of a ddt\_lcm model*


---

**Description**

Summarize the output of a ddt\_lcm model

**Usage**

```
## S3 method for class 'ddt_lcm'
summary(object, burnin = 3000, relabel = TRUE, be_quiet = FALSE, ...)
```

**Arguments**

object	a "ddt_lcm" object
burnin	number of samples to discard from the posterior chain as burn-ins. Default is 3000.
relabel	If TRUE, perform post-hoc label switching using the Equivalence Classes Representatives (ECR) method to solve non-identifiability issue in mixture models. If FALSE, no label switching algorithm will be performed.



be\_quiet        If TRUE, do not print information during summarization. If FALSE, print label switching information and model summary.

...             Further arguments passed to each method

### Value

an object of class "summary.ddt\_lcm"; a list containing the following elements:

tree\_map the MAP tree of "phylo4d" class

tree\_Sigma the tree-structured covariance matrix associated with tree\_map

response\_probs\_summary, class\_probs\_summary, Sigma\_summary, c\_summary each is a matrix with 7 columns of summary statistics of posterior chains, including means, standard deviation, and five quantiles. In particular, for the summary of item response probabilities, each row name theta\_k,g,j represents the response probability of a person in class k to consume item j in group g

max\_llk\_full a numeric value of the maximum log-likelihood of the full model (tree and LCM)

max\_llk\_lcm a numeric value of the maximum log-likelihood of the LCM only

Z\_samples a N x total\_iters integer matrix of posterior samples of individual class assignments

Sigma\_by\_group\_samples a G x total\_iters matrix of posterior samples of diffusion variances

c\_samples a total\_iters vector of posterior samples of divergence function hyperparameter

loglikelihood a total\_iters vector of log-likelihoods of the full model

loglikelihood\_lcm a total\_iters vector of log-likelihoods of the LCM model only

setting a list of model setup information. See [ddtlcm\\_fit](#)

controls a list of model controls. See [ddtlcm\\_fit](#)

data the input data matrix

### See Also

Other ddt\_lcm results: [print.ddt\\_lcm\(\)](#), [print.summary.ddt\\_lcm\(\)](#)

### Examples

```
# load the result of fitting semi-synthetic data with 1000 (for the sake of time) posterior samples
data(result_diet_1000iters)
summarized_result <- summary(result_diet_1000iters, burnin = 500, relabel = TRUE, be_quiet = TRUE)
```

---

**WAIC***Compute WAIC*

---

**Description**

Compute the Widely Applicable Information Criterion (WAIC), also known as the Widely Available Information Criterion or the Watanabe-Akaike, of Watanabe (2010).

**Usage**

```
WAIC(llk_matrix)
```

**Arguments**

`llk_matrix` a  $N \times S$  matrix, where  $N$  is the number of individuals and  $S$  is the number of posterior samples

**Value**

a named list

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